High order methods for computing axisymmetric plasma equilibria

A. Pataki, L. Greengard, NYU, J. Freidberg, A. Cerfon, MIT

We present a high order numerical method for computing equilibria of axisymmetric plasmas solving the Grad-Shafranov equation of ideal MHD. Current solvers typically use finite element or finite difference methods, and have difficulties calculating derivatives with high-order accuracy. This is due, in part, to the need for one-sided approximations near the domain boundary and, in part, to the cancellation errors inherent in numerical differentiation. Our solver aims to eliminate these difficulties by using a mix of integral equation and spectral methods, and thus can calculate all first and second derivatives with the same high convergence rate without the need for additional grid points. In brief, a change of variables is used to transform the leading differential operator into the standard two dimensional Laplacian. A conformal map is then constructed to pose the problem as a Poisson problem in the unit disk, where a fast spectral solver based on separation of variables is applied. Since most interesting equilibrium calculations involve a nonlinear plasma profile and require an eigenvalue computation, a nonlinear outer iteration is used to arrive at the final solution. We have successfully demonstrated the procedure for ITER-like and spherical tokamak plasma profiles and typically achieve 10+ digit accuracy in the solution and its derivatives in under a minute on single CPU core on a 512 by 512 grid.