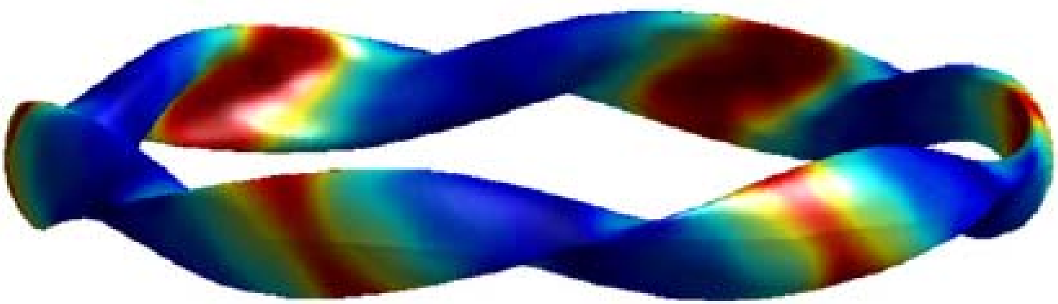
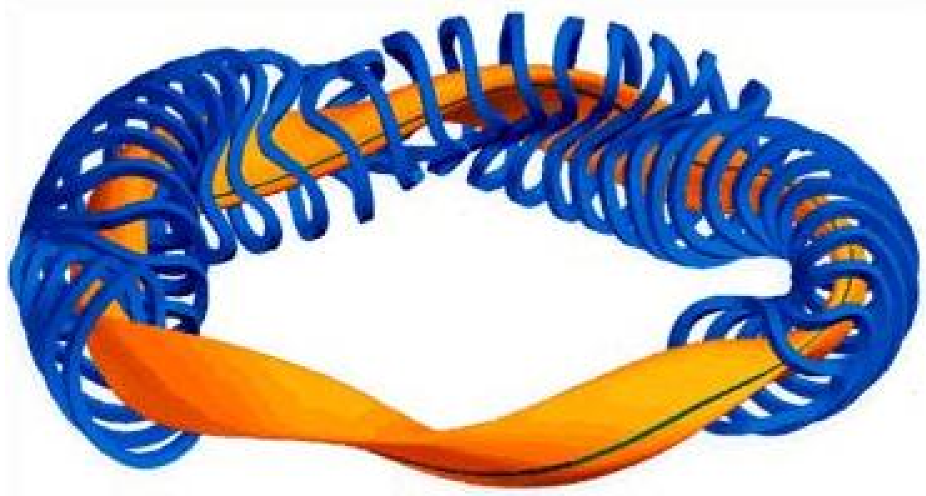
Flow, current, & electric field in omnigenous stellarators





Matt Landreman
with Peter J Catto
MIT Plasma Science & Fusion Center
Oral 2O4 – Sherwood Fusion Theory Meeting
Tuesday May 3, 2011

Supported by U.S. D.o.E.

Preview

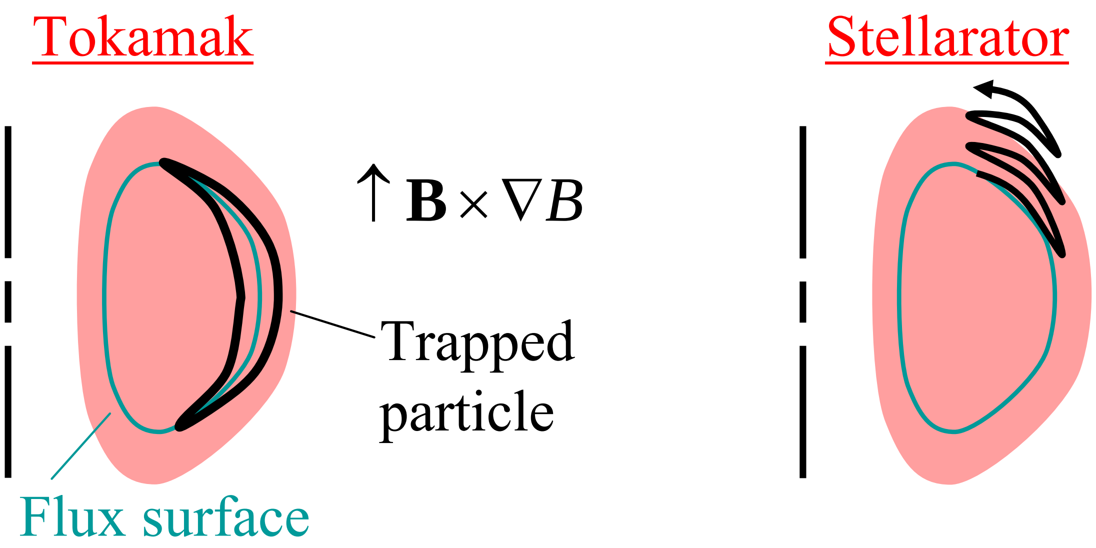
W7-X can be – and any reactor must be – nearly omnigenous.

Omnigenous stellarators:

- More general than quasisymmetric devices.
- **B** has nice properties.
- Formulae for current & flow simplify dramatically.
- Universal radial electric field.

Landreman and Catto, PPCF **53**, 035106 (2011).

Omnigenity = no unconfined orbits.



- Unconfined α particles can damage plasma-facing components.

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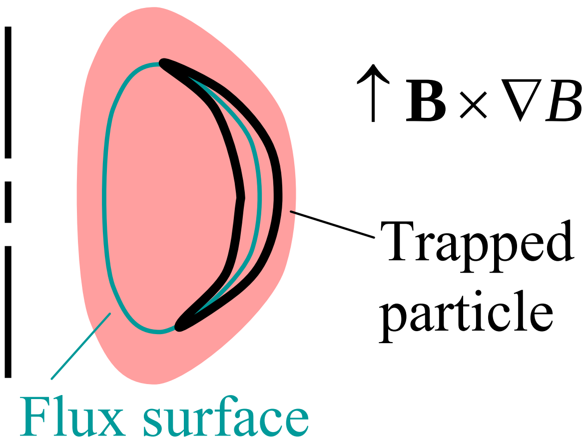
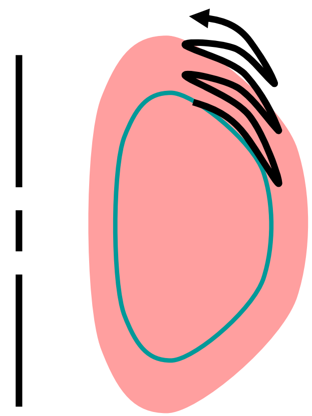
Tokamak



Stellarator



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For a reactor, then, a stellarator must be nearly *omnigenous*:

$$0 = \Delta \psi \text{ per bounce} = \oint_{\text{bounce}} \left(\mathbf{v}_d \cdot \nabla \psi \right) dt \quad \text{for all } \mu.$$

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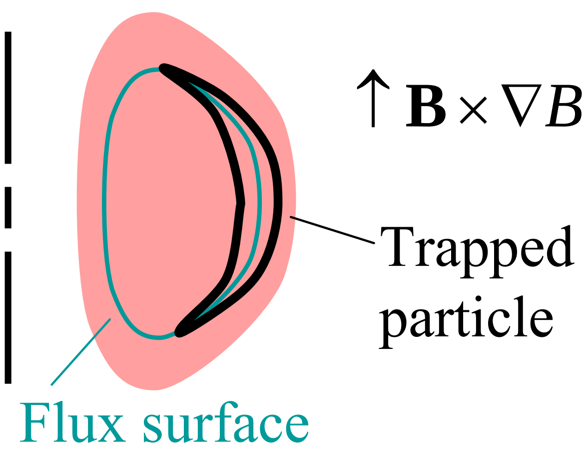

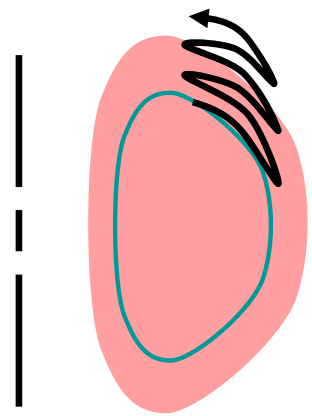

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Equivalent definition:

J is a flux function,

where $J = \oint v_{\|}\, d\ell$

is the longitudinal invariant.

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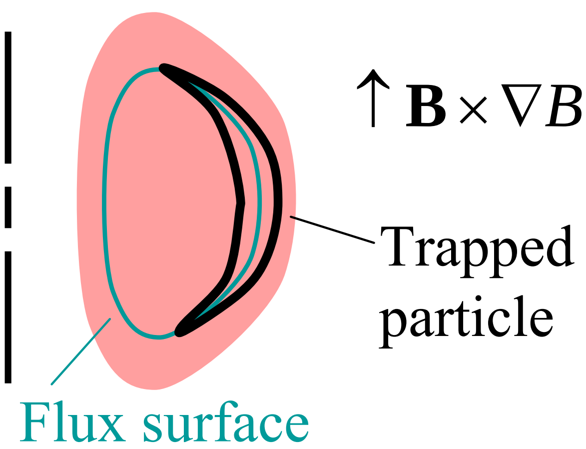

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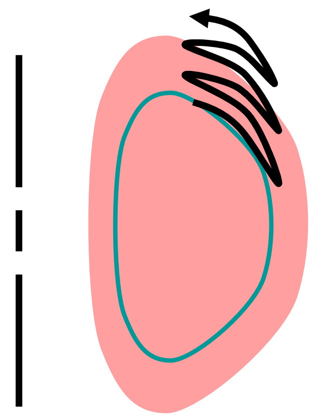

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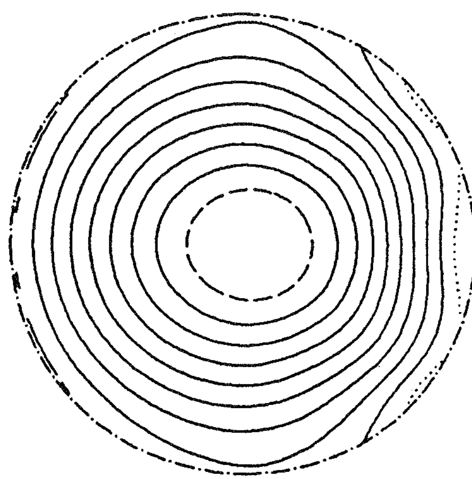
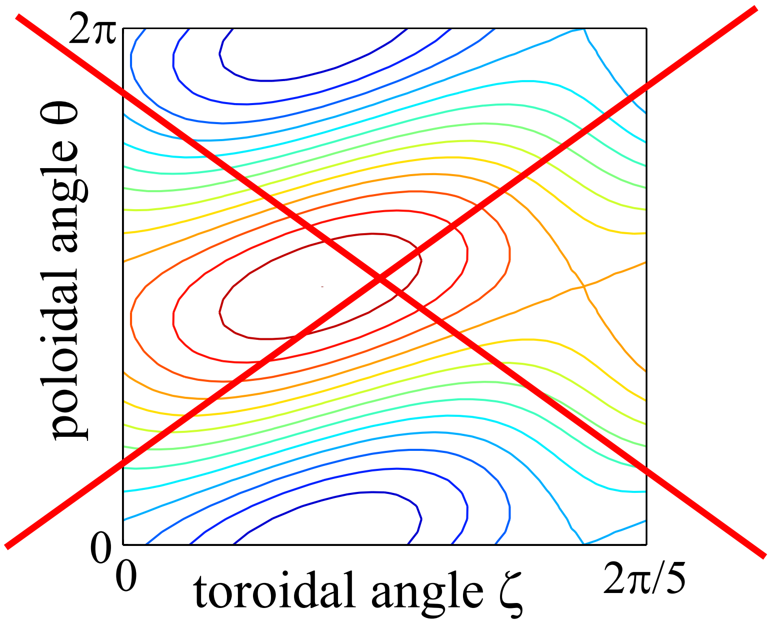


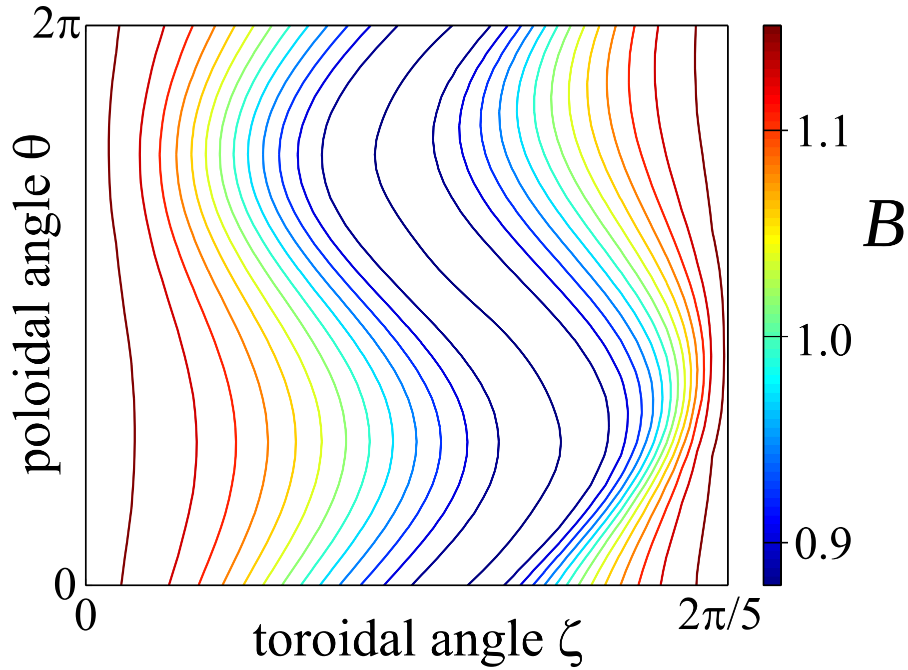
J contours for W7-X in $\left(\sqrt{\psi}, \theta\right)$ polar coordinates

Omnigenity places strong constraints on B.

If $\oint (\mathbf{v}_d \cdot \nabla \psi)\, dt = 0$, then

- All B contours link the torus toroidally, poloidally, or both.





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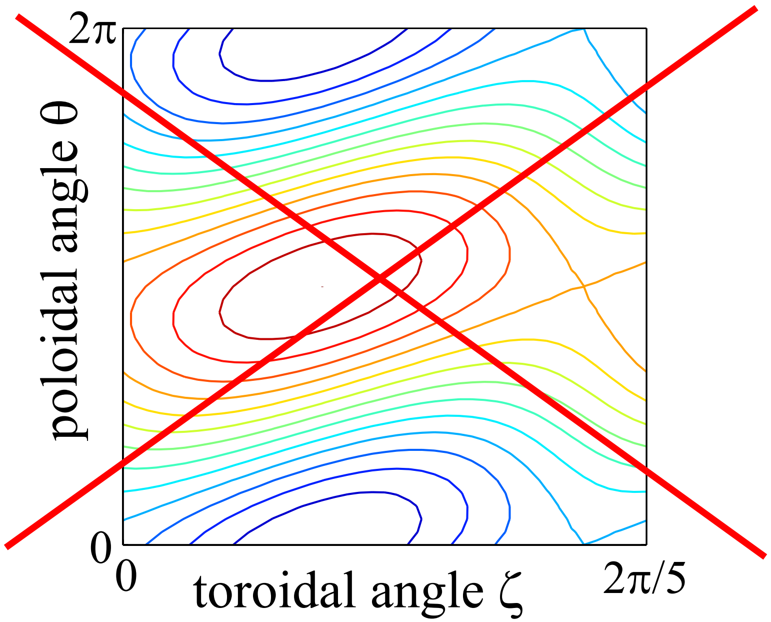
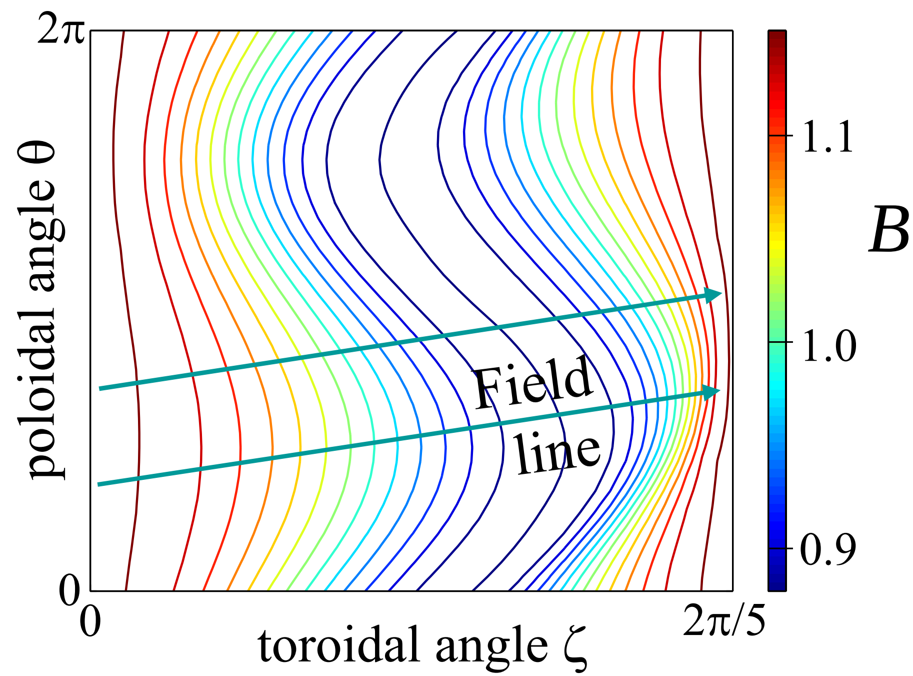
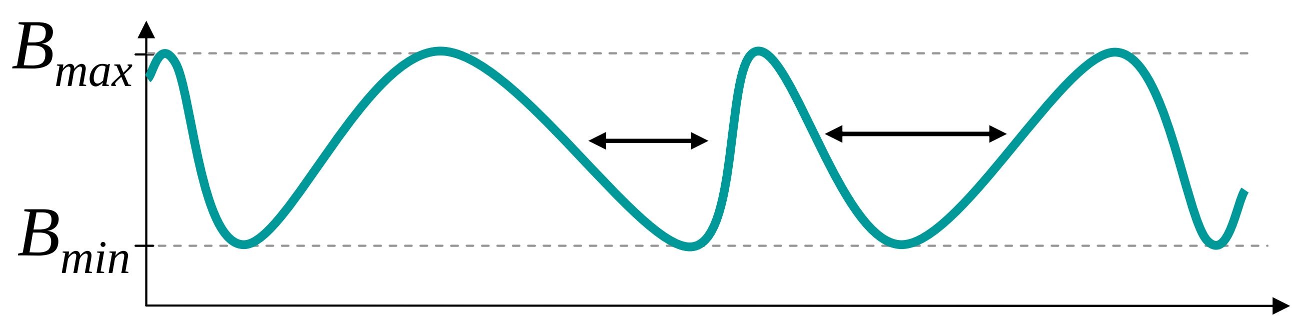




- Each maximum & minimum of *B* along a field line is the same:



One way to achieve omnigenity is *quasisymmetry.*

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Ignorable coordinate in the guiding-center Lagrangian $\Rightarrow$ Tokamak-like trajectories.

Neoclassical formulae simplify dramatically. Explicit non-numerical forms possible:

Quasisymmetry: $$\langle j_{||} B \rangle = -4.8\sqrt{\varepsilon} q \left(\frac{dp_i}{d\psi} + \frac{dp_e}{d\psi} - 0.74 n_e \frac{dT_e}{d\psi} - 1.17 n_e \frac{dT_i}{d\psi} \right) \frac{MG + NI}{M - qN}$$

Pytte & Boozer PoF (1981), Boozer PoF (1983)

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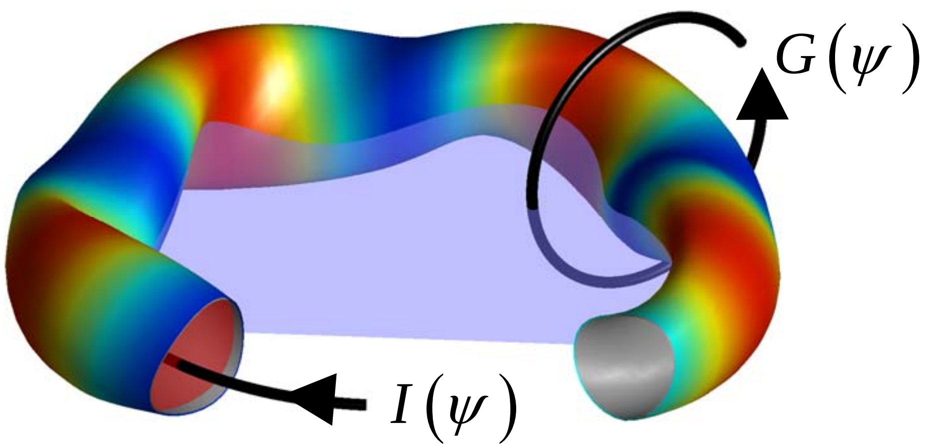
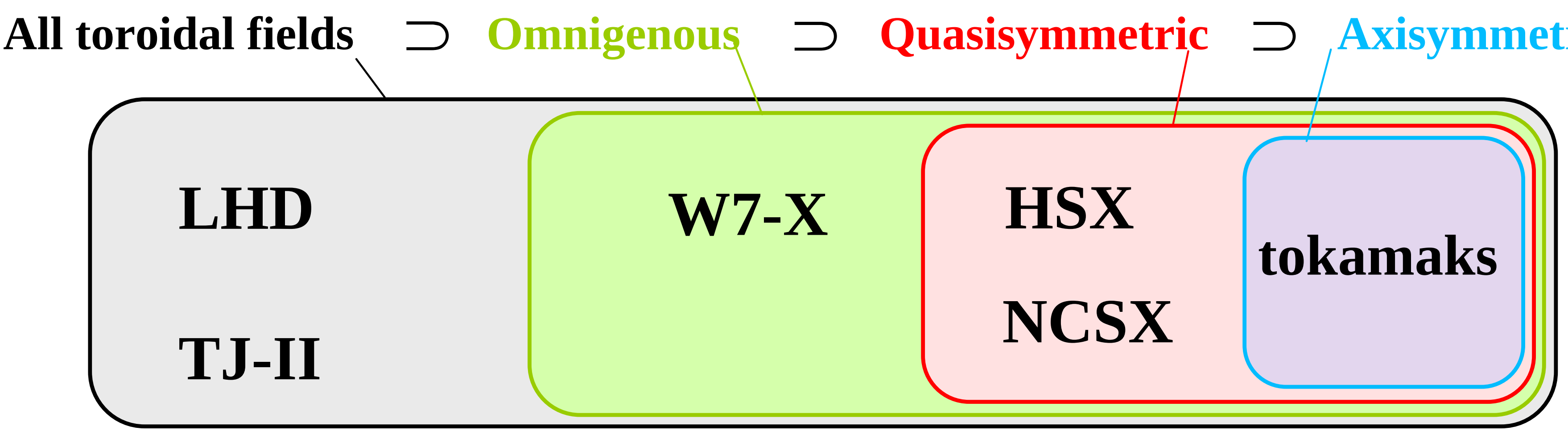
where $G(\psi) =$ poloidal current outside the flux surface,

$I(\psi) =$ toroidal current inside the flux surface

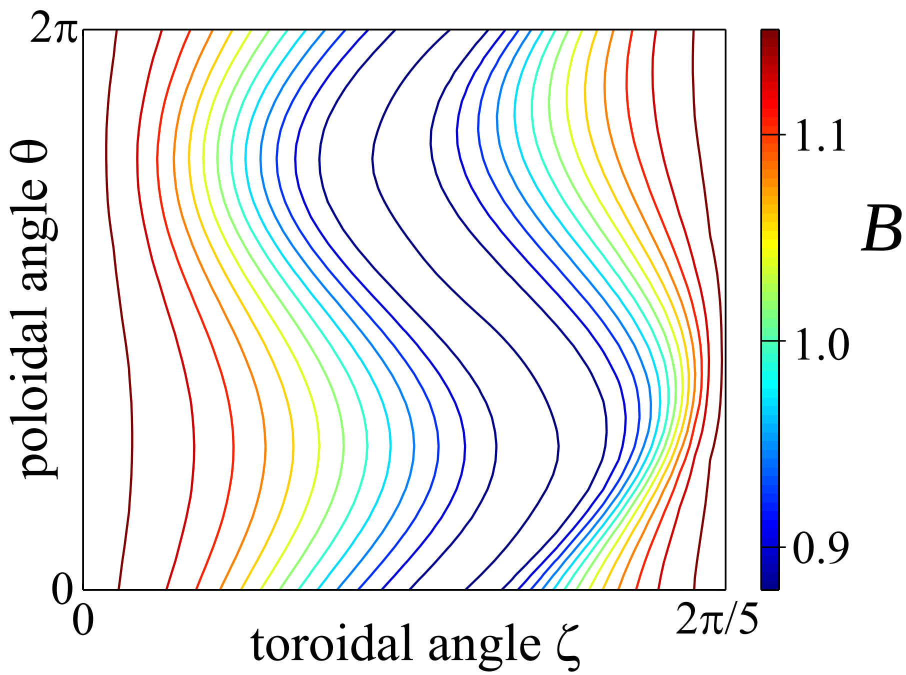


Omnigenity is more general than quasisymmetry.

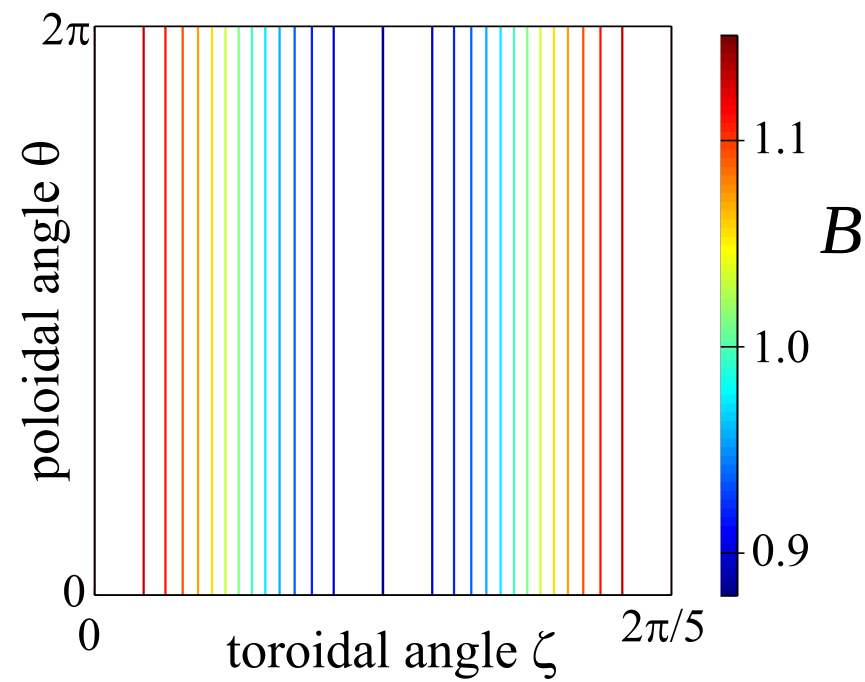
Cary & Shasharina, *PoP* (1997), *PRL* (1997)



Omnigenity: *B* contours may be curved



Quasisymmetry: *B* contours must be straight



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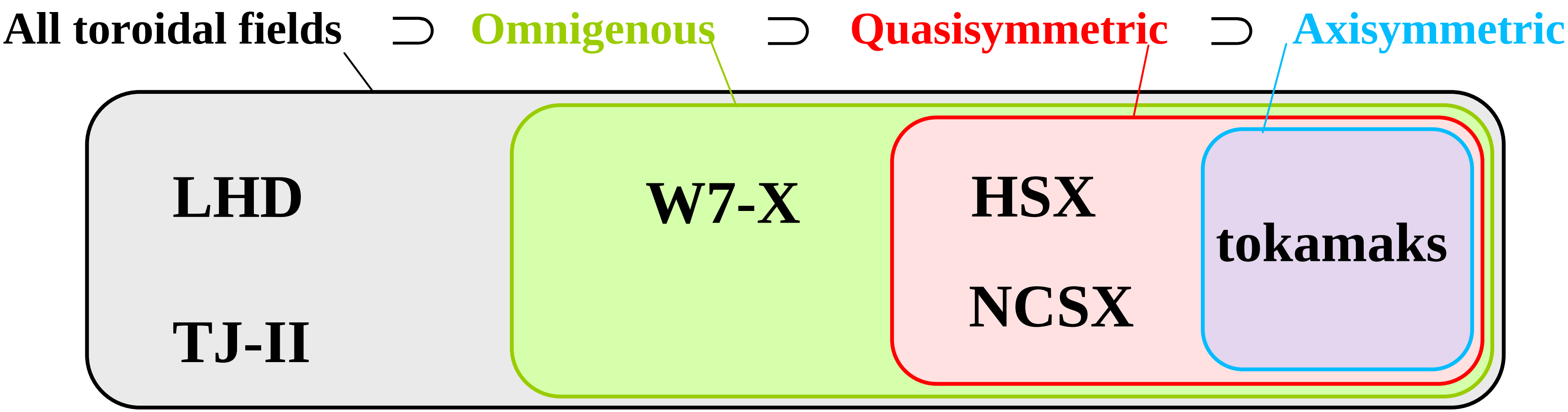

Quasisymmetry is more restrictive than omnigenity and does no more to reduce α-particle loss or neoclassical transport.

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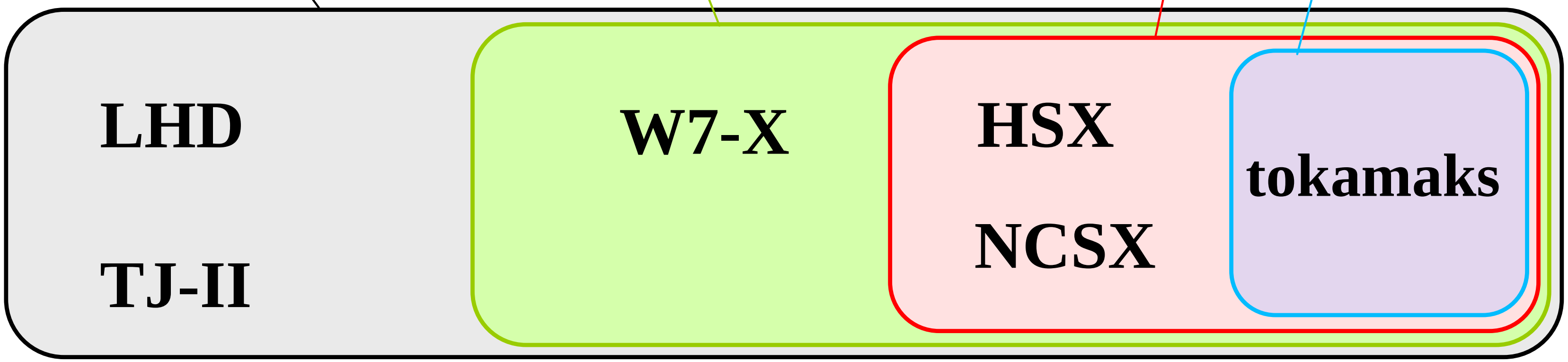
All toroidal fields $\supset$ **Omnigenous** $\supset$ **Quasisymmetric** $\supset$ **Axisymmetric**



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Quasisymmetric fields **may** have reduced instability & turbulent transport:

- Flow speeds comparable to $\upsilon_{\mathrm{th,i}}$ are only permitted in quasisymmetric fields. (*Helander, PoP 2007*)
- Flows are clamped to a weakly sheared neoclassical value except in quasisymmetry, so quasisymmetric fields permit larger flow shear. (*Helander & Simakov, PRL 2008*)

The quasisymmetry helicity (M, N) can be generalized to omnigenity.

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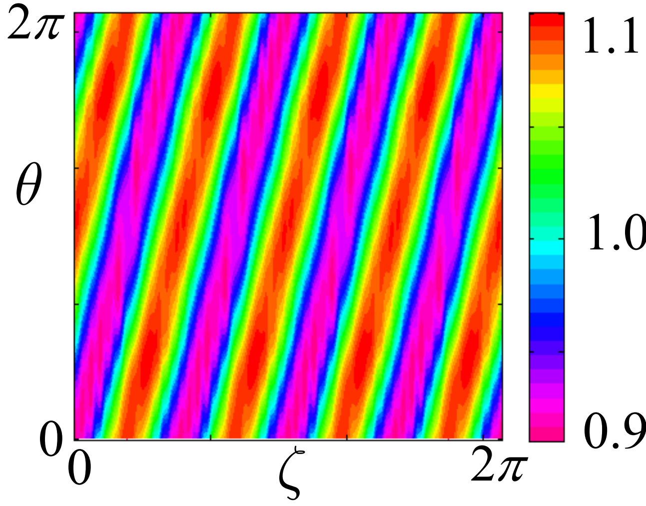
e.g. HSX:
$M = 1$, $N = 4$



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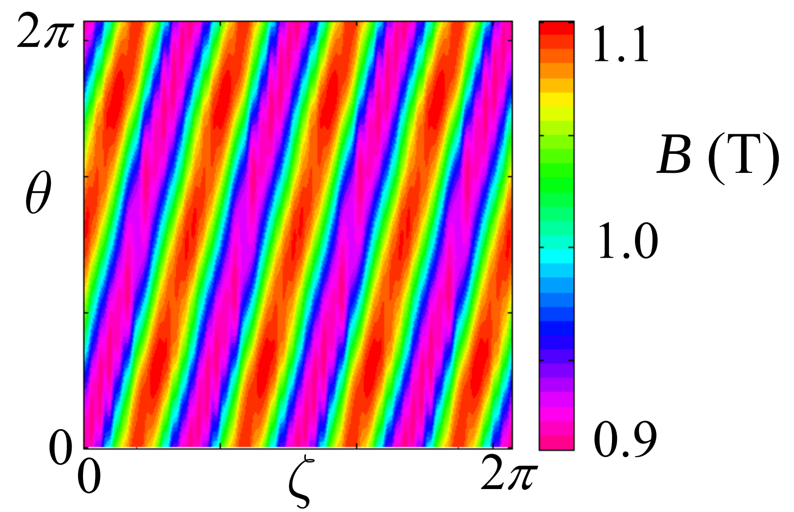

New geometric consequence of omnigenity:

By applying Ampère's Law to a B contour on a flux surface,

$$\oint \mathbf{B} \cdot d\mathbf{r} = \frac{4\pi}{c} \times \underbrace{\left(\text{enclosed current}\right)}_{MG + NI}$$

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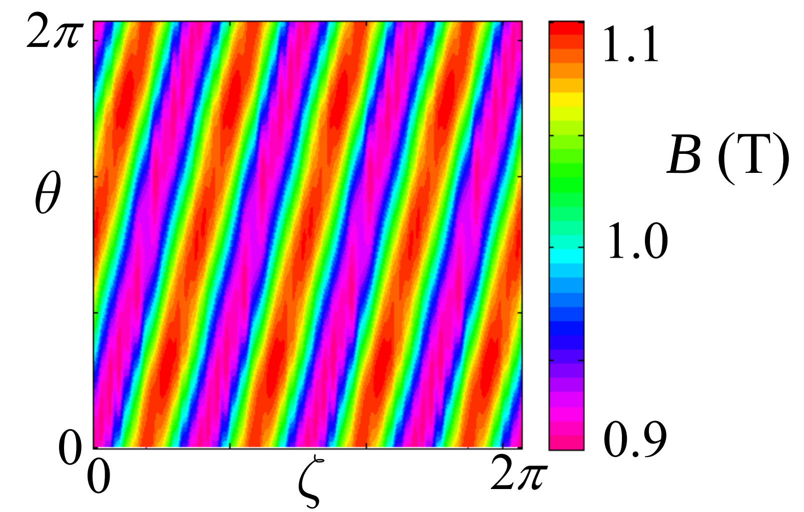

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$$\Rightarrow \quad \frac{\mathbf{B}\times\nabla\psi\cdot\nabla B}{\mathbf{B}\cdot\nabla B} = \frac{2q}{c}\left(\frac{MG+NI+H}{M-qN}\right) \quad \text{where } \left\langle H\right\rangle = 0.$$

Current in an omnigenous plasma is described by a concise, explicit, analytical formula.

$$
\underbrace{j_{\|} = 3.3 \frac{f_t q B}{\langle B^2 \rangle} \left(\frac{NI + MG}{qN - M} \right) \left(\frac{dp_e}{d\psi} + \frac{dp_i}{d\psi} - 0.74 n_e \frac{dT_e}{d\psi} - 1.17 n_e \frac{dT_i}{d\psi} \right) + \frac{2q}{B(qN - M)} \left(\frac{dp_e}{d\psi} + \frac{dp_i}{d\psi} \right) \left[\left(1 - \frac{B^2}{\langle B^2 \rangle} \right) (NI + MG)}_{\text{Tokamak result with } G \to -(NI+MG)/(qN-M)} + W \right]
$$

$$
W = \frac{2B^2}{q} (qG + I) \times \int_0^{\zeta} \frac{d\zeta'}{B'^3} \left(N \frac{\partial B'}{\partial \theta} + M \frac{\partial B'}{\partial \zeta} \right)
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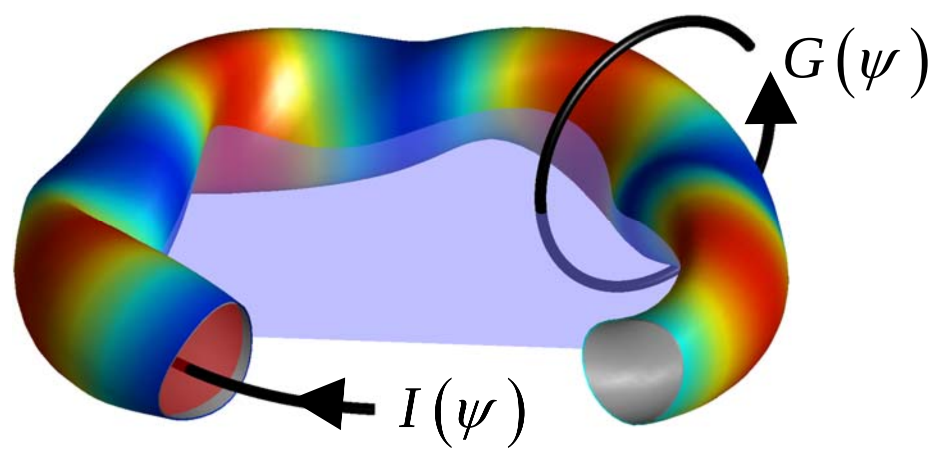
$I(\psi)$ and $G(\psi)$ are the toroidal & poloidal currents.



Bootstrap j can vanish if $M = 0$.

Subbotin *et al*, NF **46**, 921 (2006), Helander & Nührenberg, PPCF **51**, 055004 (2009).

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B contours should close poloidally $(M = 0)$ to minimize $\left\langle j_{\|}B\right\rangle$.

Flow in an omnigenous plasma is described by a concise, explicit, analytical formula.

$$\underbrace{V_{\|i} = -1.17 \frac{2qB}{e\left\langle B^2 \right\rangle} \frac{dT_i}{d\psi} \frac{\left(NI + MG\right)}{\left(qN - M\right)} + \frac{2q}{B}\left(\frac{d\Phi}{d\psi} + \frac{1}{en}\frac{dp_i}{d\psi}\right)\frac{\left(NI + MG\right)}{\left(qN - M\right)}}_{\text{Tokamak result with } G \to -\left(NI + MG\right)/\left(qN - M\right)}\frac{\left(NI + MG + W\right)}{\left(qN - M\right)}$$

$$W = \frac{2B^2}{q}\left(qG + I\right) \times \int_0^{\zeta} \frac{d\zeta'}{B'^3}\left(N\frac{\partial B'}{\partial \theta} + M\frac{\partial B'}{\partial \zeta}\right)$$

$$\left\langle W \right\rangle = 0$$

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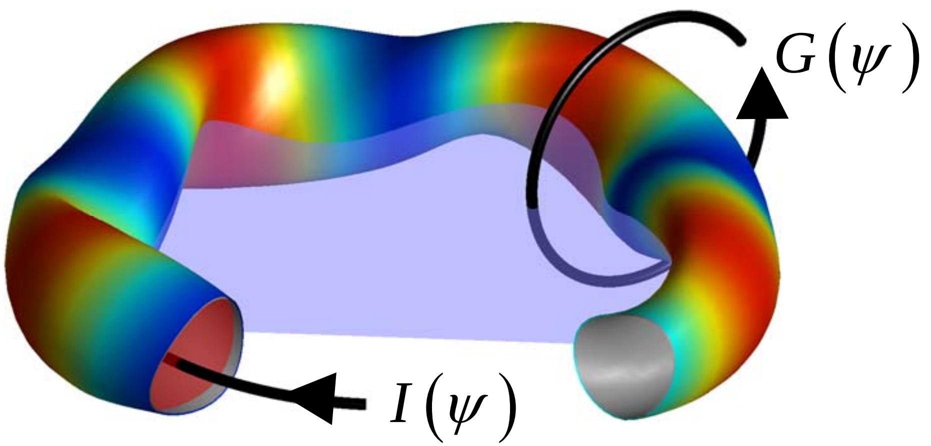

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<u>Non-quasisymmetric stellarators:</u>

- Neoclassical radial current depends on E_r.
- $\langle \mathbf{j}_{\text{neoclassical}} \cdot \nabla \psi \rangle \gg \langle \mathbf{j}_{\text{turbulence}} \cdot \nabla \psi \rangle$.

(*Helander & Simakov, Contrib. Plasma Phys. 2010*)

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<u>Tokamaks & quasisymmetric stellarators:</u>

- Radial fluxes of ions and electrons are always equal, regardless of E_r ("intrinsic ambipolarity")

(*Helander & Simakov, PRL 2008*)

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<u>Omnigenous stellarators:</u>

$$\langle \mathbf{j} \cdot \nabla \psi \rangle = \left(Zen_i \frac{d\Phi}{d\psi} + T_i \frac{dn_i}{d\psi} - 0.17 n_i \frac{dT_i}{d\psi} \right) \left\langle \left(\text{departure from quasisymmetry} \right)^2 \right\rangle$$

E_r is determined by ambipolarity.

Non-quasisymmetric stellarators:

- Neoclassical radial current depends on E_r.
- $\langle \mathbf{j}_{\text{neoclassical}} \cdot \nabla\psi \rangle \gg \langle \mathbf{j}_{\text{turbulence}} \cdot \nabla\psi \rangle$.

(*Helander & Simakov, Contrib. Plasma Phys. 2010*)

$\Rightarrow$ You can solve for E_r using $\langle \mathbf{j}_{\text{neoclassical}} \cdot \nabla\psi \rangle = 0$.

Omnigenous stellarators:

$$\left\langle \mathbf{j}\cdot\nabla\psi \right\rangle = \left(Zen_i \frac{d\Phi}{d\psi} + T_i \frac{dn_i}{d\psi} - 0.17 n_i \frac{dT_i}{d\psi} \right) \left\langle \left(\text{departure from quasisymmetry} \right)^2 \right\rangle$$

Universal result:

$$\boxed{\frac{d\Phi}{d\psi} = \frac{1}{Ze}\left(-\frac{T_i}{n_i}\frac{dn_i}{d\psi} + 0.17\frac{dT_i}{d\psi} \right)}$$

Totally independent of the details of **B**.

Summary: omnigenity is a useful limit.

- More general than quasisymmetry.
- Relevant (at least for insight and code benchmarking) to W7-X and to any reactor.
- Concise, explicit, analytical formulae for **j**, **V**, and E_r.
- Poloidally closed B contours $\Rightarrow$ zero bootstrap current.
- For non-quasisymmetric **B**, E_r is determined explicitly:

$$\frac{d\Phi}{d\psi} = \frac{1}{Ze}\left(-\frac{T_i}{n_i}\frac{dn_i}{d\psi} + 0.17\frac{dT_i}{d\psi}\right)$$ independent of the **B** geometry.

Landreman and Catto, PPCF **53**, 035106 (2011).